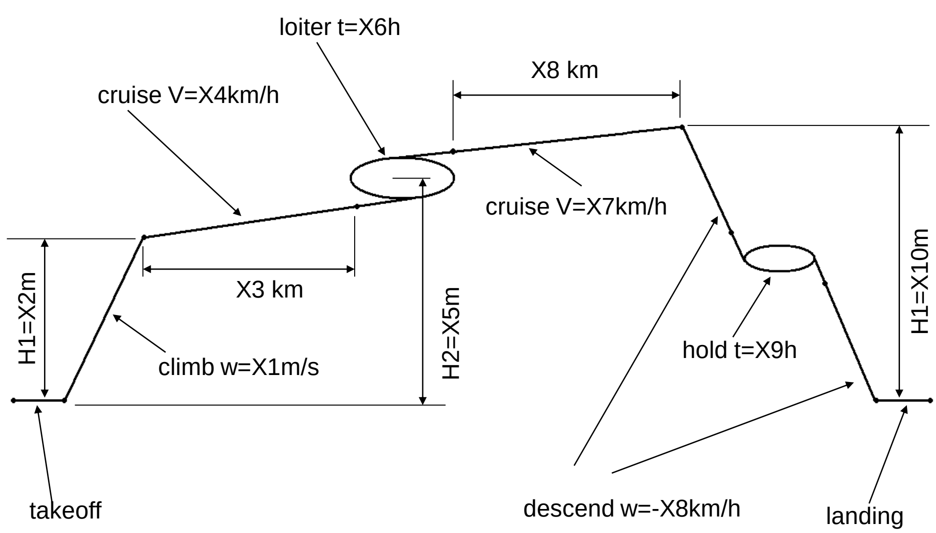
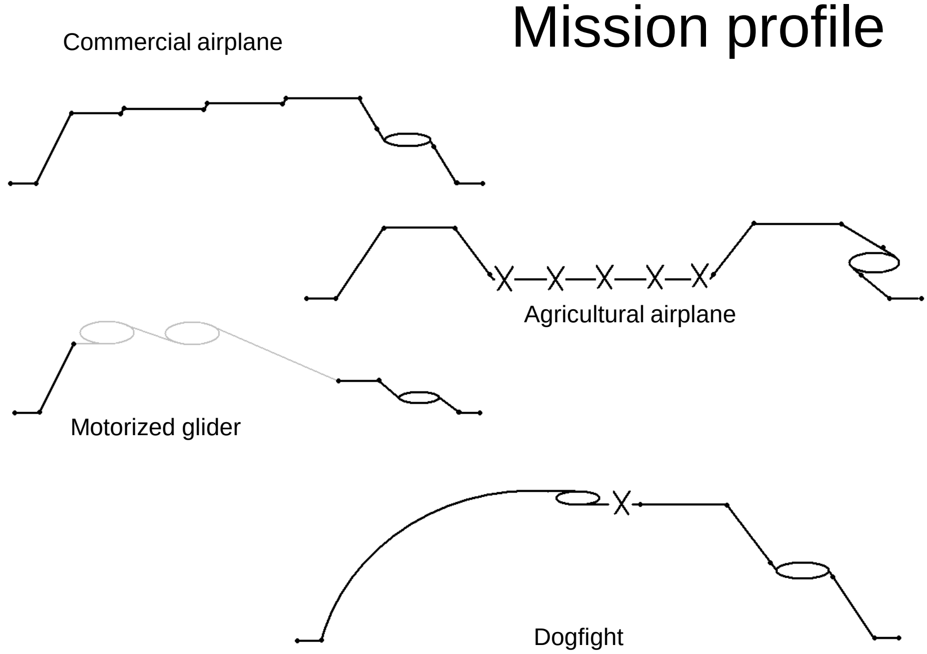
Airplane sizing

Mission profile



Mission profile



$$W_{TO} = W_c + W_f + W_p + W_e$$

$$W_p = W_{exp} + W_{nexp}$$

W_{TO} – takeoff weight

W_c – crew weight

W_f – fuel weight

W_p – payload weight

W_e – empty weight

W_{exp} – expendable payload weight

W_{nexp} – nonexpendable payload weight

Airplanes expending their payloads in flight

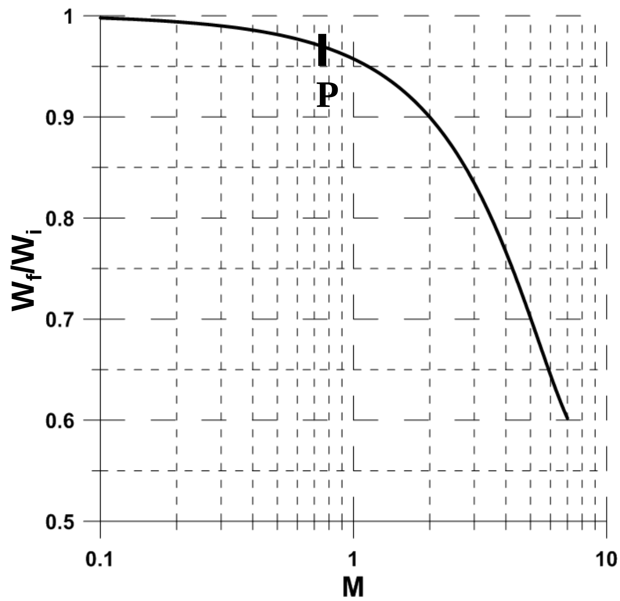
1) Assume the takeoff weight
2) Calculate weight after takeoff
3) Calculate weight after climb (ATTENTION: weight before this phase is equal to the weight after previous phase)
4) Calculate weight after cruise (ATTENTION: as above)
5) Calculate weight after task execution (ATTENTION1: as above, ATTENTION2: airplane can release a parto of its payload)
6) Calculate weight after return cruise (ATTENTION: as above)
7) Calculate weight after holding (ATTENTION: as above)
8) Calculate weight after landing and taxi (ATTENTION: as above)
9) Calculate an empty weight (ATTENTION: subtract weights of crew, remaining payload, fuel reserve weight and trapped fuel weight from weight after landing)
10) Calculate realistic empty weight from $W_e/W_{TO}=f(W_{TO})$ plot
11) Calculate the difference between weights calculated in points 9) i 10)
12) Modify takeoff weight

Takeoff

Takeoff = engine ignition and warmup + taxi + takeoff run + + climbing to prescribed altitude

$$0,97 \leq (W_1 / W_{TO})_f \leq 0,975$$

Climb and acceleration



Corke

Cruise - Breguet's equations (Imperial)

1) $$\mathbf{R = \frac{V}{C} \cdot \frac{L}{D} \cdot \ln(W_n / W_{n+1})}$$

2) $$\mathbf{R = \frac{\eta}{C} \cdot \frac{L}{D} \cdot \ln(W_n / W_{n+1})}$$

Ad 1 jet airplane
Ad 2 propeller airplane
L – Lift
D – Drag
η - propeller acceleration
C – specific fuel consumption in $lb_{fuel}/h/lb_{thrust}$ for jets
$lb_{fuel}/h/SHP$ for propeller driven airplanes

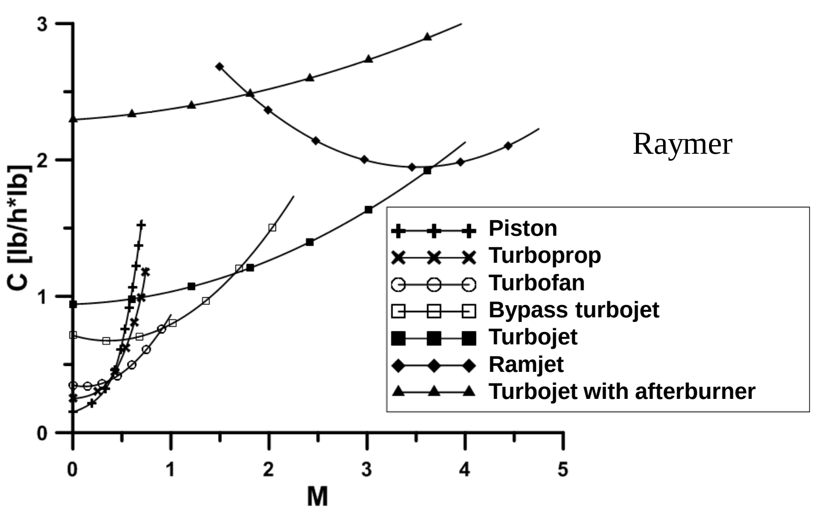
Cruise - Breguet's equations (SI+Polish symbols)

1) $$R = \frac{V}{C \cdot g} \cdot \frac{C_z}{C_x} \cdot \ln(W_n / W_{n+1})$$

2) $$R = \frac{\eta}{C \cdot g} \cdot \frac{C_z}{C_x} \cdot \ln(W_n / W_{n+1})$$

Ad 1 jet airplane
Ad 2 propeller airplane
Cz – Lift coefficient (C_L in English)
Cx – Drag coefficient (C_D in English)
η - propeller acceleration
C – specific fuel consumption in kg/h/kN for jets
kg/h/kW for propeller
driven airplanes

$$\eta \approx 0{,}8$$

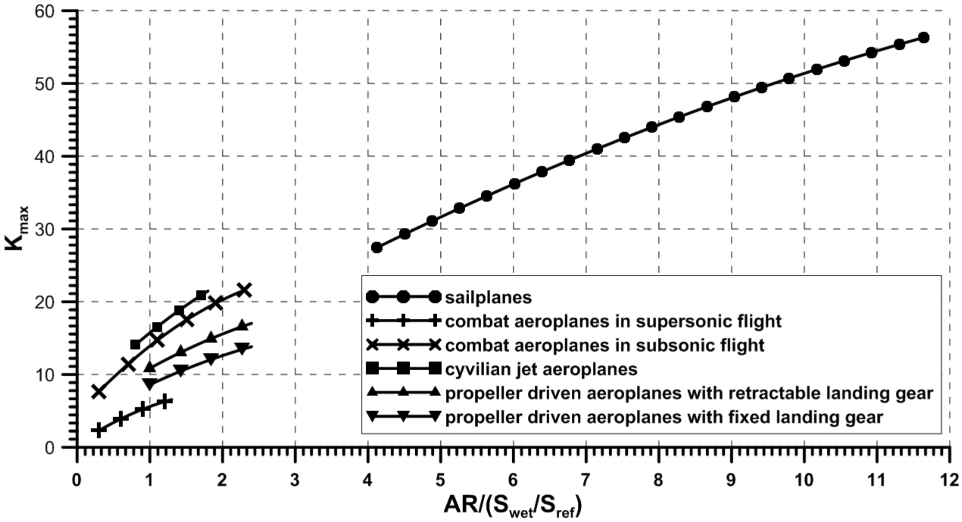


Raymer

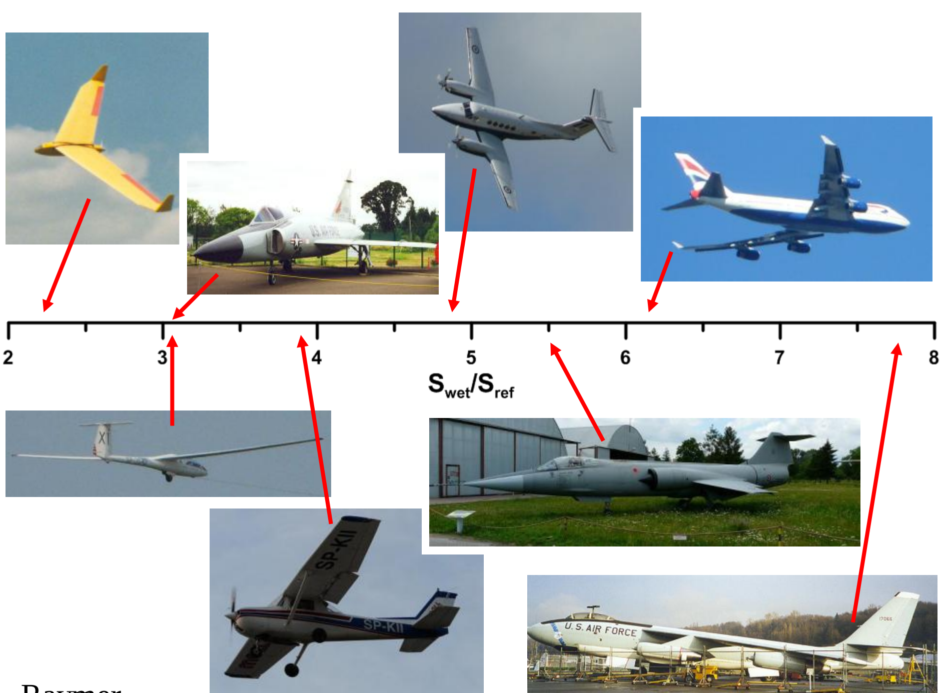
or

E. Cichosz „Charakterystyka i zastosowanie napędów"

$$\frac{C_L}{C_D} \approx 0{,}94 \cdot \left(\frac{C_L}{C_D}\right)_{max}$$



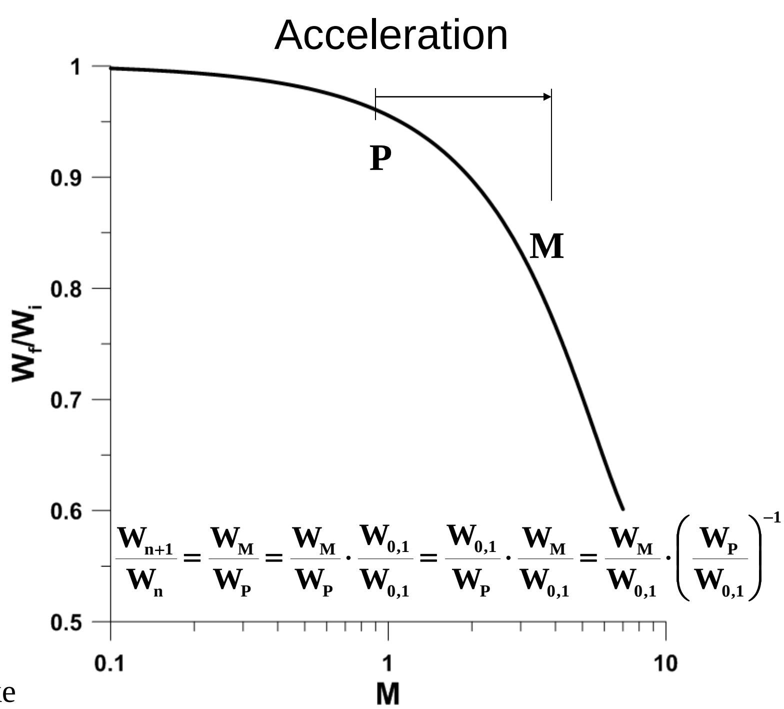
Raymer



Raymer

Raymer

	B-47	Avro Vulcan
Sr reference area	**1430**	**3446**
Sw wetted area	**11300**	**9600**
Wing span	**116**	**90**
Sw/Sr	**7,9**	**2,8**
AR	**9,4**	**3,0**
AR*Sr/Sw	**1,2**	**1,1**
L/D max	**17,2**	**17,0**



$$\frac{W_{n+1}}{W_n} = \frac{W_M}{W_P} = \frac{W_M}{W_P} \cdot \frac{W_{0,1}}{W_{0,1}} = \frac{W_{0,1}}{W_P} \cdot \frac{W_M}{W_{0,1}} = \frac{W_M}{W_{0,1}} \cdot \left(\frac{W_P}{W_{0,1}}\right)^{-1}$$

Corke

Combat

$$W_n - W_{n+1} = C_{max} \cdot T_{max} \cdot t$$

$$\frac{W_{n+1}}{W_n} = 1 - \frac{C_{max} \cdot T_{max} \cdot t}{W_n}$$

T- thrust
t - time

Loiter

$$t = \frac{1}{C \cdot g} \cdot \frac{C_L}{C_D} \cdot \ln(W_n / W_{n+1})$$

$$t = \frac{\eta}{C \cdot g} \cdot \frac{C_L}{C_D} \cdot \frac{1}{V} \cdot \ln(W_n / W_{n+1})$$

e.g. reconnaissance aircraft

Return cruise

$$R = \frac{V}{C \cdot g} \cdot \frac{C_L}{C_D} \cdot \ln(W_n / W_{n+1})$$

$$R = \frac{\eta}{C \cdot g} \cdot \frac{C_L}{C_D} \cdot \ln(W_n / W_{n+1})$$

W is lower so it is advantageous to fly higher. Airplane should be re-trimmed what increases the drag.

Holding before landing

$$t = \frac{1}{C \cdot g} \cdot \frac{C_L}{C_D} \cdot \ln(W_n / W_{n+1})$$

$$t = \frac{\eta}{C \cdot g} \cdot \frac{C_L}{C_D} \cdot \frac{1}{V} \cdot \ln(W_n / W_{n+1})$$

Landing

$$0{,}97 \le (W_{n+1} / W_n)_f \le 0{,}975$$

Airplanes expending their payloads in flight

1) **Assume the takeoff weight**
2) **Calculate weight after takeoff**
3) **Calculate weight after climb (ATTENTION: weight before this phase is equal to the weight after previous phase)**
4) **Calculate weight after cruise (ATTENTION: as above)**
5) **Calculate weight after task execution (ATTENTION1: as above, ATTENTION2: airplane can release a part of its payload)**
6) **Calculate weight after return cruise (ATTENTION: as above)**
7) **Calculate weight after holding (ATTENTION: as above)**
8) **Calculate weight after landing and taxi (ATTENTION: as above)**
9) **Calculate an empty weight (ATTENTION: subtract weights of crew, remaining payload, fuel reserve weight and trapped fuel weight from weight after landing)**
10) **Calculate realistic empty weight from $W_e/W_{TO}=f(W_{TO})$ plot**
11) **Calculate the difference between weights calculated in points 9) i 10)**
12) **Modify takeoff weight**

Airplanes not expending their payloads in flight

$$W_{TO} = W_{crew} + W_{payload} + W_{fuel} + W_{empty}$$

$$W_{TO} = W_{cr} + W_{pay} + \frac{W_f}{W_{TO}} W_{TO} + \frac{W_e}{W_{TO}} W_{TO}$$

$$W_{TO} = \frac{W_{cr} + W_{pay}}{1 - \frac{W_f}{W_{TO}} - \frac{W_e}{W_{TO}}}$$

$$W_L + \frac{W_F}{1{,}06} = W_{TO}$$

$$\frac{W_L}{W_{TO}} + \frac{W_F}{1{,}06 \cdot W_{TO}} = 1$$

$$W_F / W_{TO} = 1{,}06(1 - W_L / W_{TO})$$

$$(W_L / W_{TO})_f = \frac{W_L}{W_{TO}} \cdot \frac{W_1}{W_1} \cdot \frac{W_2}{W_2} \cdot \ldots \cdot \frac{W_m}{W_m} = \frac{W_1}{W_{TO}} \cdot \frac{W_2}{W_1} \cdot \ldots \cdot \frac{W_L}{W_m}$$

$W_{1 \ldots m}$ – airplane weights after each phase of the flight

Wing and power loading estimation

Takeoff

$$s_{TO} = 20,9(TOP) + 87\sqrt{(TOP)(T/W)}$$

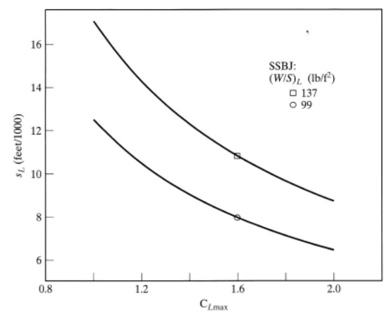
$$TOP = \left(\frac{W}{S}\right)_{TO} \frac{1}{C_{L\,max}} \left(\frac{W}{T}\right)_{TO} \frac{1}{\sigma}$$

ang. - $C_L = C_Z$ – pol.

$$\sigma = \rho_{TO} / \rho_{SL}$$

Landing

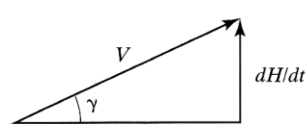
$$LP = \left(\frac{W}{S}\right)\frac{1}{\sigma C_{L\max}}$$



$$s_L = 118(LP) + 400$$

Climb

$$\frac{dH}{dt} = V\sin\gamma = \frac{P_s / W}{1 + \frac{V}{g}\frac{dV}{dH}}$$



$$P_s = V(T - D)$$

	MIL-C5011A Military	FAR Part 23 Civil	FAR Part 25 Commercial
Gear Up, AEO	500 fpm at SL	300 fpm at SL	—
Gear Up, OEI	100 fpm at SL	—	3% at V_{CL}
Gear Down, OEI	—	—	0.5% at V_{CL}

AEO = all engines operating
OEI = one engine inoperative

for dV/dH=0

$$\sin\gamma = \frac{T - D}{W} = G$$

Ps – excess power
G - gradient

Climb

$$qC_{D0} + \left(\frac{W}{S}\right)^2 \frac{1}{q\pi Ae} - \left(\frac{T}{W} - G\right)\frac{W}{S} = 0$$

Climb

$$\frac{W}{S} = \frac{\left(\frac{T}{W} - G\right) \pm \sqrt{\left(\frac{T}{W} - G\right)^2 - 4\frac{C_{D0}}{\pi Ae}}}{\frac{2}{q\pi Ae}}$$

$$\frac{T}{W} \geq G + 2\sqrt{\frac{C_{D0}}{\pi Ae}}$$

Range – propeller driven airplane

$$R = \frac{\eta}{C \cdot g} \cdot \frac{C_L}{C_D} \cdot \ln(W_n / W_{n+1})$$

Maximum range will be reached for C_L reflecting maximum gliding ratio:

$$\frac{W}{S} = q\sqrt{\pi \cdot A \cdot e \cdot C_{D0}}$$

Range – jet airplane

$$R = \frac{V}{C \cdot g} \cdot \frac{C_L}{C_D} \cdot \ln(W_n / W_{n+1})$$

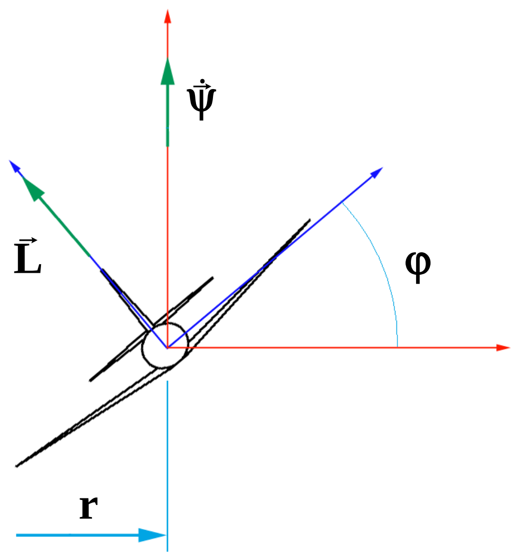
Maximum range will be reached when VC_L/C_D achieves the maximum

$$\frac{W}{S} = q\sqrt{\frac{C_{D0}\pi A e}{3}}$$

Turn

$$\begin{cases} W = L\cos\varphi \\ \dfrac{mV^2}{r} = L\sin\varphi \end{cases}$$

$$\frac{1}{\cos\varphi} = \frac{L}{W} = n$$



Turn

$$\psi = \frac{g\sqrt{n^2 - 1}}{V}$$

$$\frac{W}{S} = \frac{qC_{L\max}}{\sqrt{\left(\frac{\dot{\psi}V}{g}\right)^2 + 1}}$$

Sustained turn

$$T = D$$

Sustained turn

$$\left(\frac{W}{S}\right)^2 \frac{n^2}{q\pi Ae} - \frac{T}{W}\frac{w}{s} + qC_{D0} = 0$$

Sustained turn

$$\frac{W}{S} = \frac{\frac{T}{W} \pm \sqrt{\left(\frac{T}{W}\right)^2 - \frac{4n^2 C_{D0}}{\pi A e}}}{\frac{2n^2}{q\pi A e}}$$

$$\frac{T}{W} \geq 2n\sqrt{\frac{C_{D0}}{\pi A e}}$$

Sustained turn

$$n_{max} = \sqrt{\frac{q\pi A e}{W/S}\left(\left(\frac{T}{W}\right)_{max} - \frac{qC_{D0}}{W/S}\right)}$$

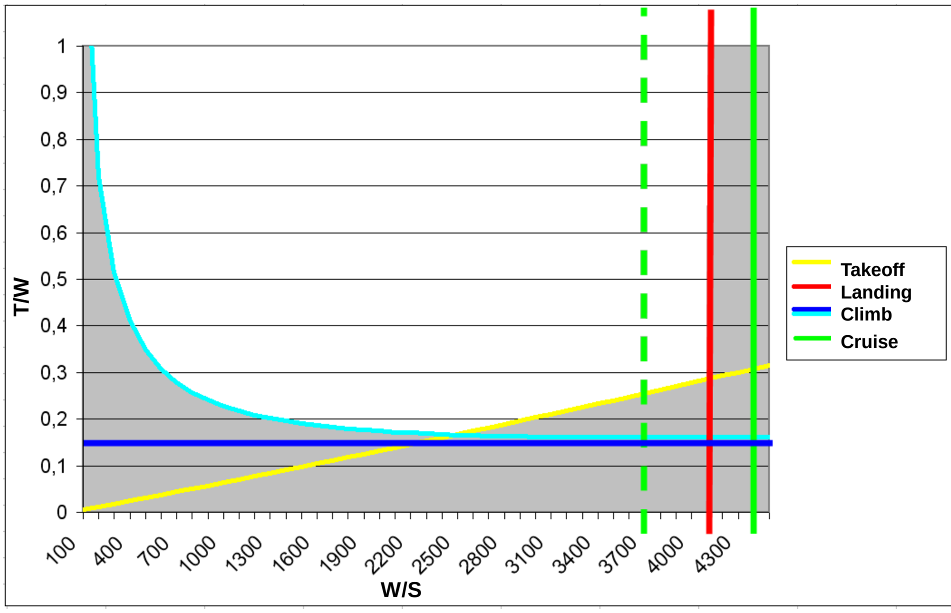
$$\dot{\psi}_{max} = \frac{g\sqrt{\frac{q\pi A e}{W/S}\left(\left(\frac{T}{W}\right)_{max} - \frac{qC_{D0}}{W/S}\right) - 1}}{V}$$

Maximum altitude

$$n = 1$$

$$\frac{W}{S} = qC_{L\max} = C_{L\max}\frac{\rho_{h\max}V^2}{2}$$

Takeoff	$s_{TO} = 20{,}9(TOP) + 87\sqrt{(TOP)(T/W)}$ $TOP = \left(\frac{W}{S}\right)_{TO}\frac{1}{C_{L\max}}\left(\frac{W}{T}\right)_{TO}\frac{1}{\sigma}$
Landing	$s_L = 118(LP) + 400$ $LP = \left(\frac{W}{S}\right)\frac{1}{\sigma C_{L\max}}$
Climb	$\frac{W}{S} = \frac{\left(\frac{T}{W} - G\right) \pm \sqrt{\left(\frac{T}{W} - G\right)^2 - 4\frac{C_{D0}}{\pi Ae}}}{\frac{2}{q\pi Ae}}$ $\frac{T}{W} \geq G + 2\sqrt{\frac{C_{D0}}{\pi Ae}}$
Acceleration	$\frac{W}{S} = \frac{q}{n}\sqrt{\pi AeC_{D0}}$
Range	$\frac{W}{S} = q\sqrt{\frac{C_{D0}\pi Ae}{3}}$ or $\frac{W}{S} = q\sqrt{\pi \cdot A \cdot e \cdot C_{D0}}$
Turn	$\frac{W}{S} = \frac{qC_{L\max}}{\sqrt{\left(\frac{\psi V}{g}\right)^2 + 1}}$
Sustained turn	$\frac{W}{S} = \frac{\frac{T}{W} \pm \sqrt{\left(\frac{T}{W}\right)^2 - \frac{4n^2C_{D0}}{\pi Ae}}}{\frac{2n^2}{q\pi Ae}}$ $\frac{T}{W} \geq 2n\sqrt{\frac{C_{D0}}{\pi Ae}}$
Maximum altitude	$\frac{W}{S} = qC_{L\max} = C_{L\max}\frac{\rho_{h\max}V^2}{2}$



Example

Assumptions:

Takeoff weight	33100kg
Fuel weight	9700kg
Cruise airspeed	780km/h na 10000m
Minimum airspeed	200km/h
Aspect ratio	8
L/D	15
Takeoff & landing distance	1100m
Climb	9m/s

Takeoff

$$s_{TO} = 20{,}9(TOP) + 87\sqrt{(TOP)(T/W)}$$

$$TOP = \left(\frac{W}{S}\right)_{TO} \frac{1}{C_{Lmax}} \left(\frac{W}{T}\right)_{TO} \frac{1}{\sigma}$$

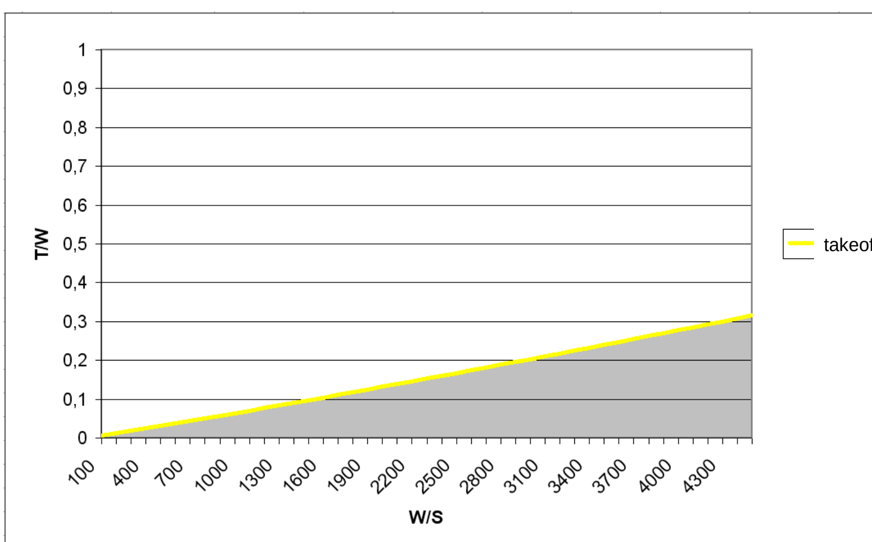
Imperial

$$\frac{T}{W} = \frac{0{,}133 \cdot \left(\frac{W}{S}\right) \cdot \frac{1}{\sigma \cdot C_{Lmax}}}{S_{TO} - 3{,}834\sqrt{\frac{W}{\sigma \cdot S \cdot C_{Lmax}}}}$$

SI

Takeoff

$$C_{Lmax} = 2$$



Landing

$$s_L = 118(LP) + 400$$

$$LP = \left(\frac{W}{S}\right)\frac{1}{\sigma C_{Lmax}}$$

Imperial

$$\frac{W}{S} = \frac{(S_L - 122)\sigma C_{Lmax}}{0,75}$$

SI

Landing

$$\frac{W}{S} = \frac{(S_L - 122)\sigma C_{Lmax}}{0,75}$$

$$C_{Lmax} = 2,17$$

$$\left(\frac{W}{S}\right)_L = 2920 N / m^2$$

Example

Assumptions:

Takeoff weight	**33100kg**
Fuel weight	**9700kg**
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Minimum airspeed	200km/h
Aspect ratio	8
L/D	15
Takeoff & landing distance	1100m
Climb	9m/s

Landing

$$\frac{W}{S} = \frac{(S_L - 122)\sigma C_{Lmax}}{0,75} \qquad C_{Lmax} = 2,17$$

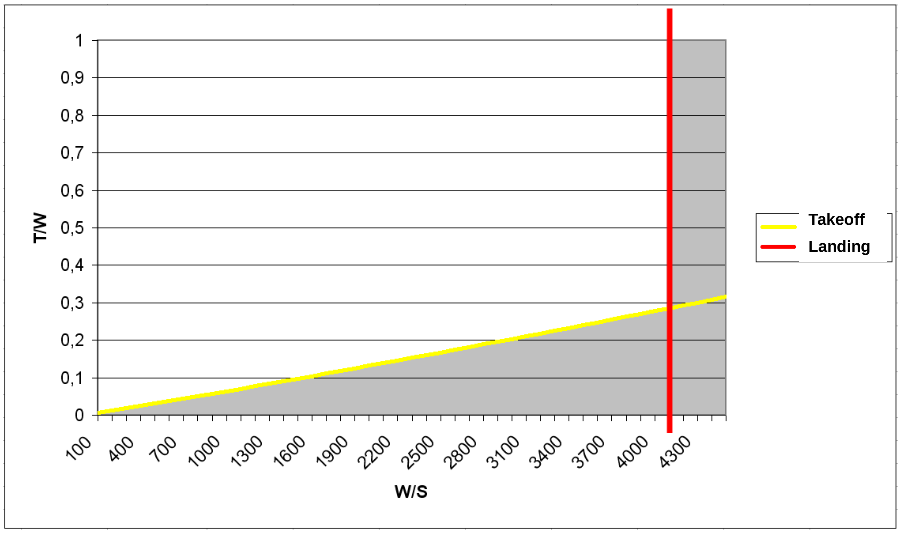
$$\left(\frac{W}{S}\right)_L = 2920 N/m^2$$

$$m_L = 33100 - 9700 = 23400 kg$$

$$S = \frac{23400 \cdot 9,81}{2920} = 79 m^2$$

$$\left(\frac{W}{S}\right)_{TO} = \frac{33100 \cdot 9,81}{79} = 4130 N/m^2$$

Takeoff and landing



Climb

$$qC_{D0} + \left(\frac{W}{S}\right)^2 \frac{1}{q\pi Ae} - \left(\frac{T}{W} - G\right)\frac{W}{S} = 0$$

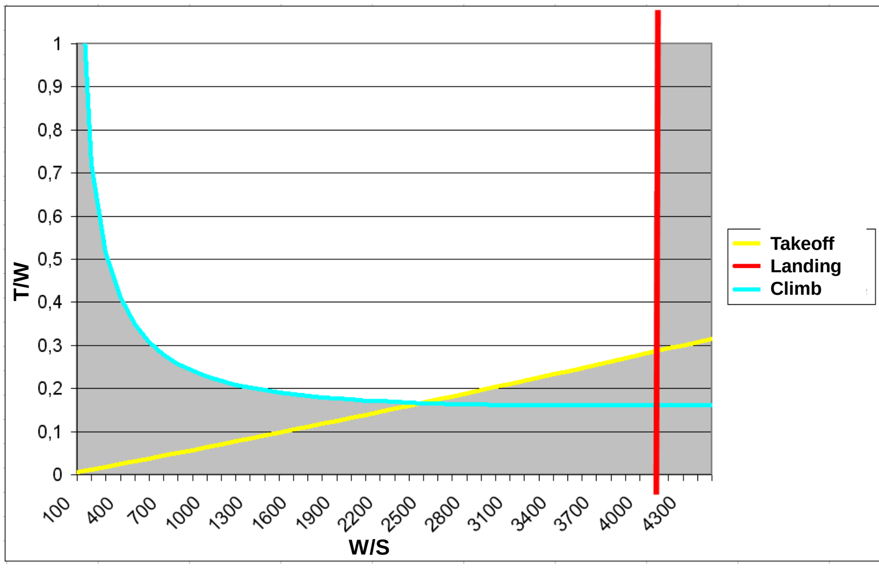
$$\frac{T}{W} = \frac{qC_{D0} + \left(\frac{W}{S}\right)^2 \frac{1}{q\pi Ae}}{\frac{W}{S}} + G$$

$$G = \frac{w_z}{V}$$

$$\left(\frac{C_L}{C_D}\right)_{max} = \frac{1}{2}\sqrt{\frac{\pi \cdot A \cdot e}{C_{D0}}}$$

Takeoff, climb and landing

V_{climb} = 350 km/h G=0,094, C_{D0} = 0,0222



Climb

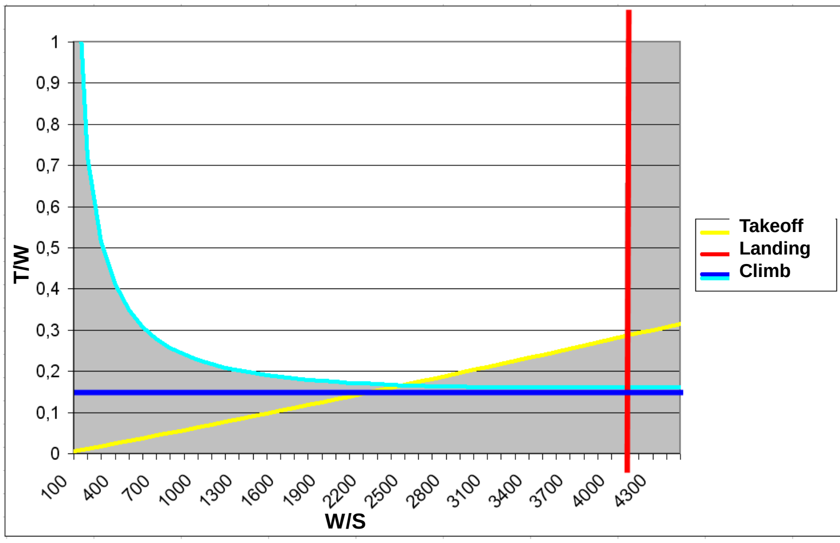
$$qC_{D0} + \left(\frac{W}{S}\right)^2 \frac{1}{q\pi Ae} - \left(\frac{T}{W} - G\right)\frac{W}{S} = 0$$

$$\frac{W}{S} = \frac{\left(\frac{T}{W} - G\right) \pm \sqrt{\left(\frac{T}{W} - G\right)^2 - 4\frac{C_{D0}}{\pi Ae}}}{\frac{2}{q\pi Ae}}$$

$$\frac{T}{W} \geq G + 2\sqrt{\frac{C_{D0}}{\pi Ae}}$$

Takeoff, climb and landing

$$\frac{T}{W} \geq G + 2\sqrt{\frac{C_{D0}}{\pi Ae}}$$

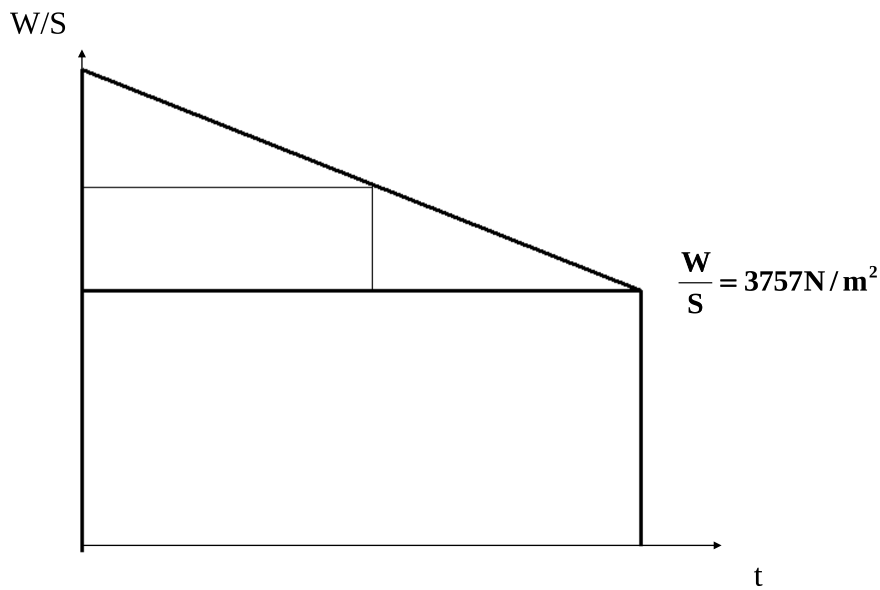


Cruise

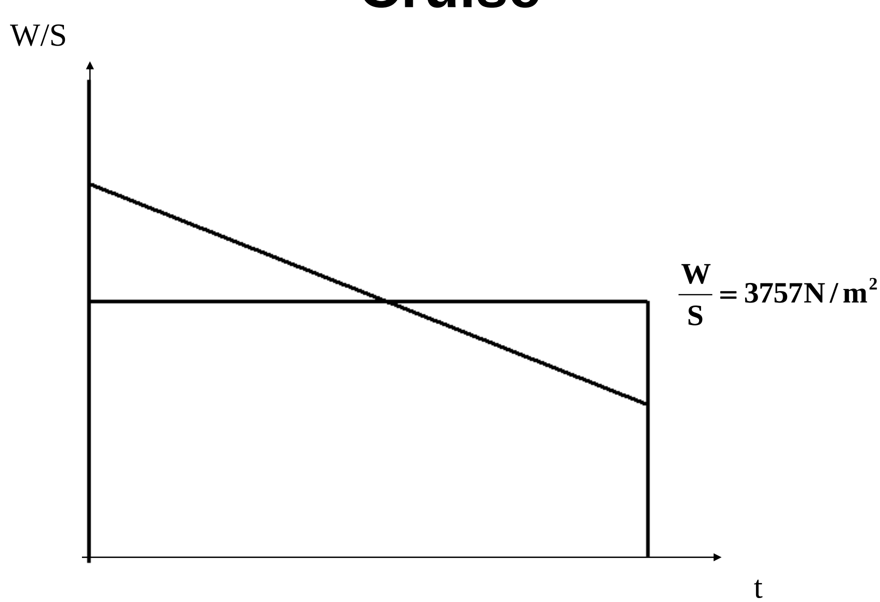
$$\frac{W}{S} = q\sqrt{\frac{C_{D0}\pi Ae}{3}}$$

$$\frac{W}{S} = 3757N/m^2$$

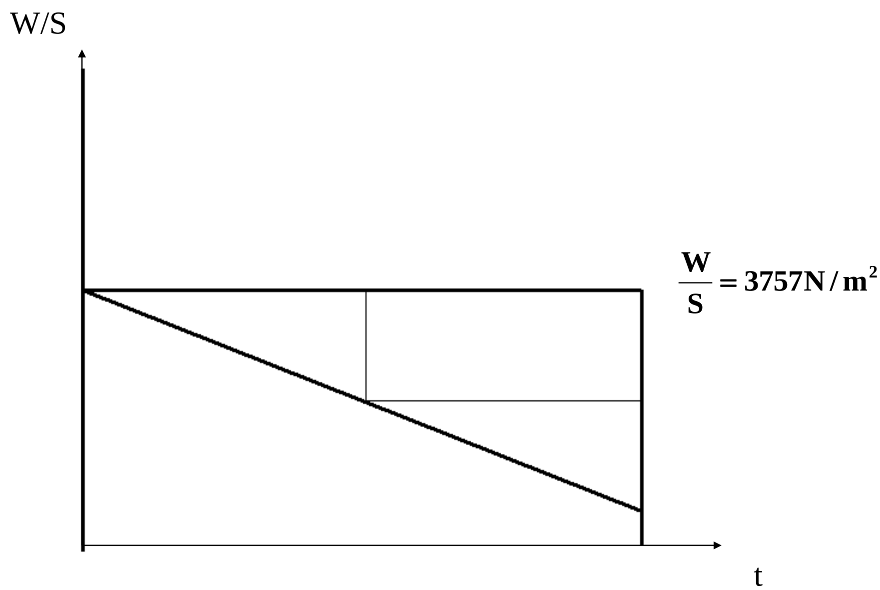
Cruise



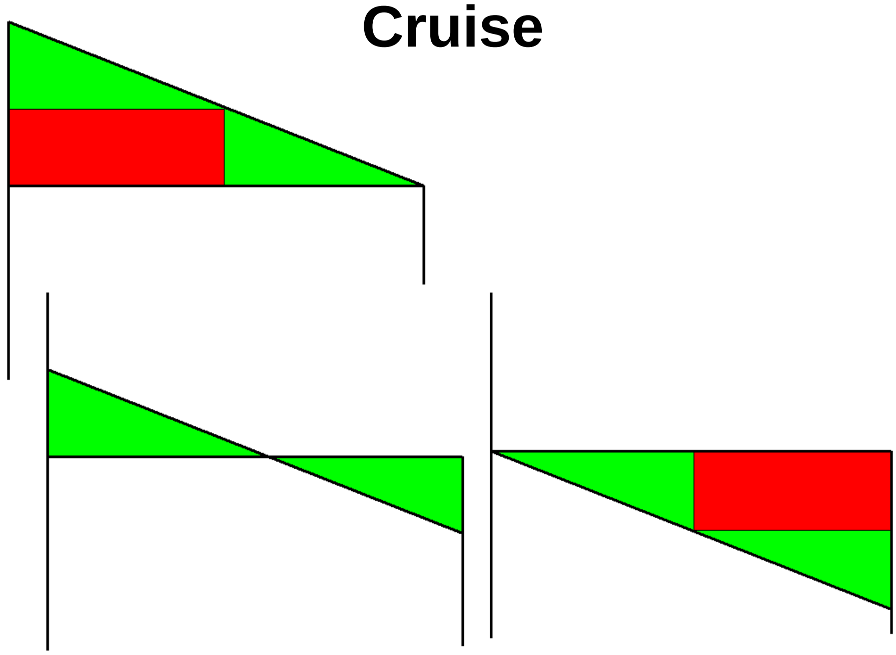
Cruise



Cruise



Cruise



Example

Assumptions:

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Fuel weight	**9700kg**
Cruise airspeed	780km/h na 10000m
Minimum airspeed	200km/h
Aspect ratio	8
L/D	15
Takeoff & landing distance	1100m
Climb	9m/s

Cruise

$$\frac{W}{S} = q\sqrt{\frac{C_{D0}\pi Ae}{3}}$$

$$\frac{W}{S} = 3757 N/m^2$$

$$m = 33100 - 4850 = 28250 kg$$

$$S = \frac{28250 * 9,81}{3757} = 74 m^2$$

$$\left(\frac{W}{S}\right)_{TO} = \frac{33100 \cdot 9,81}{74} = 4405 N/m^2$$

Design point