Project No.4

Wing optimisation

1. Introduction

The main objective of this project is to optimise the wing geometry designed at the Aircraft Design classes. The optimisation is simplified to focus on two decision variables. The optimisation can be based on the algorithm presented below. The presented algorithm illustrates optimisation based on drag at cruising conditions and assumes that wing span and mean geometric chord are the decision variables. This algorithm might be changed when needed, e.g. when one of the parameters is fixed for different reasons. In such a case, another parameter, which defines the wing geometry, must be chosen or the objective function must be modified.

2. Optimisation algorithm

Assumption:

Wing geometry is to be optimised. The optimisation assumes that wing span and mean geometric chord are the decision variables. Thus, the vector of the decision variables is as follows:

$$v = [v_0, v_1] = [b, c_g]$$

The total drug could be defined as follows:

$$D(v) = \frac{\rho V_c^2}{2} S(v)(C_{x0} + \frac{C_L^2(v)}{\pi \Lambda(v) e(v)})$$

where:

$S(v) = bc_g = v_0 v_1$ - wing area

$\Lambda(v) = \frac{b^2}{S(v)}$ - aspect ratio

$e(v) = 4.61(1 - 0.045\Lambda^{0.68}(v))\cos^{0.15}(\varphi_{LE}) - 3.1$ - Oswald's coefficient

V_c - cruise velocity

$C_L(v) = \frac{2m(v)g}{\rho S(v) V_c^2}$ - lift coefficient at cruise velocity

C_{D0} - minimum drag coefficient

ρ - air density

$m(v) = m_{bp} + m_p(v)$ - aircraft mass

$m_p(v) = 4.936 \cdot S(v)\Lambda^{0.3}(v)$ - wing mass

m_{bp} - mass of the aircraft without the wing

Also:

$C_{L,Vmin} \frac{2m(v)g}{\rho S(v) V_{min}^2}$ - lift coefficient at minimum velocity

$C_{L,\max}$ - maximum lift coefficient

Penalty method should be used to minimise the drag force. The objective function should be defined as:

$$F(v) = D(v) + K(v)$$

With the penalty function:

$$K(v) = \left[\frac{1}{C_{L,max} - C_L(V)} - \frac{1}{C_{L,max} - C_{L,Vmin}(V)}\right]^2$$

3. Computations

Computations should be started with taking the initial values of the decision variables. It should be done based on the geometrical data from the Aircraft Design Project No. 2. Wing mass should be also taken based on the mass analysis from the same project.

Let the vector of decision variables be:

$$\mathbf{X}_0 = [b_0, c_{g,0}]$$

where:

$b_0, c_{g,0}$ - initial values of the wing span and chord

Then, iterative computations with N iterations should be run as follows:

$$\mathbf{X}^q = \mathbf{X}^{q-1} + \alpha_q^* \mathbf{S}^q$$

where:

$\mathbf{S}^q$ - search vector, defined as:

$$\mathbf{S}^q = -\nabla \mathbf{F}(\mathbf{X}^{q-1})$$

$\nabla \mathbf{F}(\mathbf{X})$ - objective function gradient:

$$\nabla \mathbf{F}(X) = \begin{bmatrix} \partial F(\mathbf{X}) / \partial X_0 \\ \partial F(\mathbf{X}) / \partial X_1 \end{bmatrix}$$

α_q^* - distance in the direction of vector S which we plan to cover in the qth iteration – can be assumed to be a unit vector

Partial derivatives of the objective function can be computed numerically using the difference quotient definition, i.e.:

$$\partial F(X)/\partial x \cong \frac{F(X+\Delta x)-F(X-\Delta x)}{2\Delta x}$$

Computations should stop when the sought values cease to change significantly in the following iterations. The main objective of this project is to find the optimised values of the decision variables. It is also required that the report contains graphs of the objective function vs. definition variables and a graph which would depict the constraints (in this example the maximum lift coefficient equal to 2.5). Sample graphs are presented below:

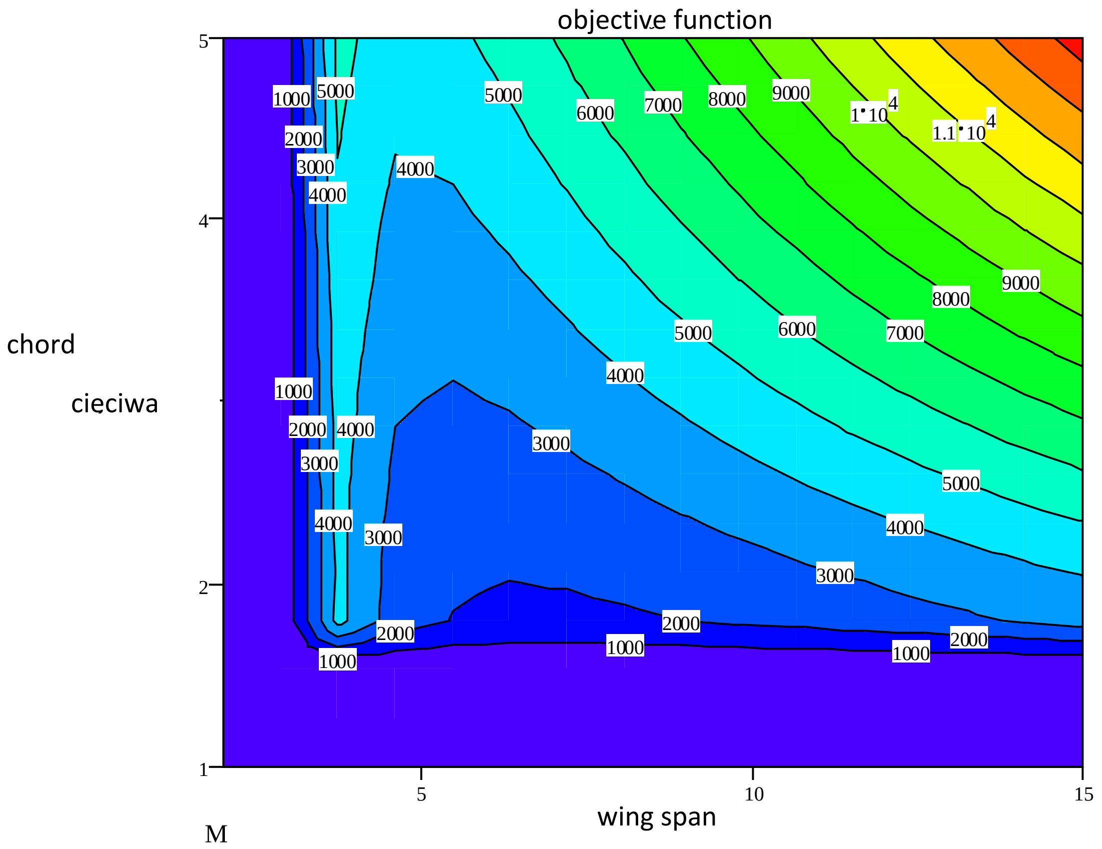

Figure 1 – Drag as a function of the wing span and chord

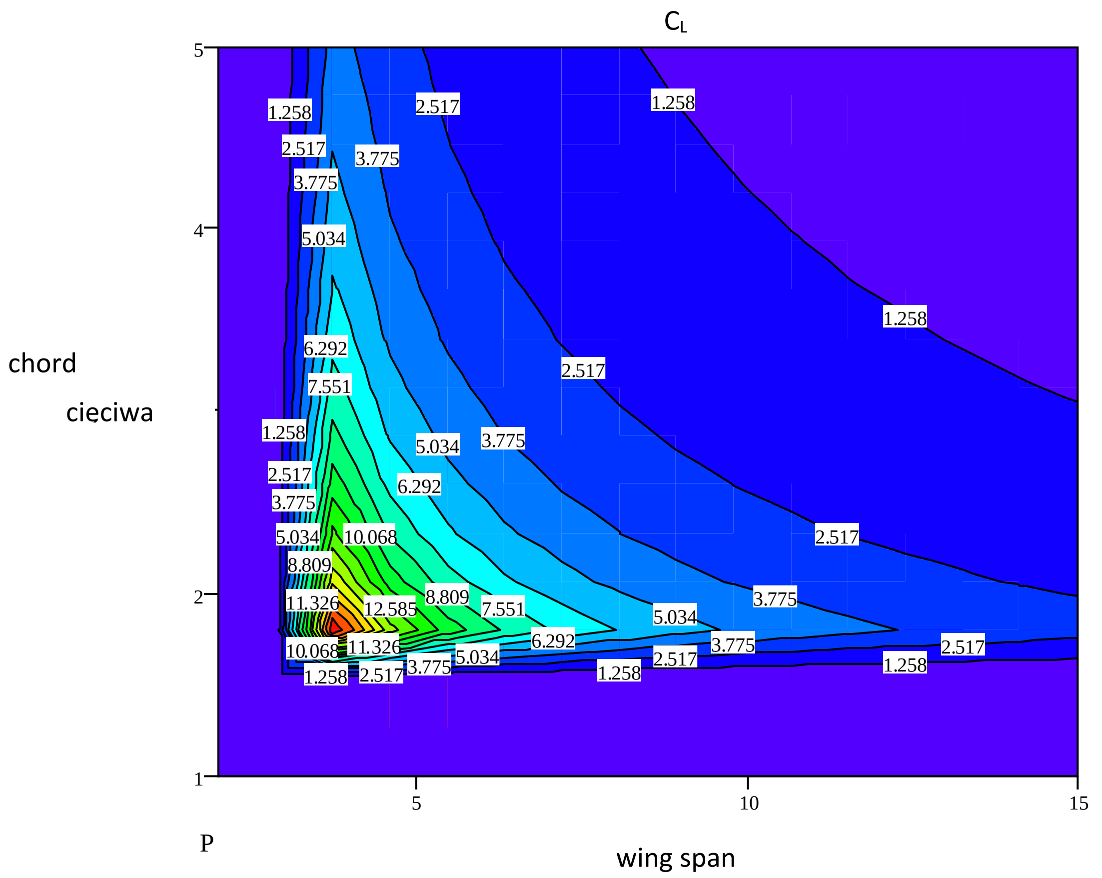

Figure 2 – Lift coefficient at minimum velocity as a function of the wing span and chord

Final remarks:

1. Penalty method is not a fully reliable method. The obtained results may depend strongly on the chosen starting point. In addition, there is a possibility of exceeding the limit value of constraints. Therefore, it is necessary to check whether this has not occurred (the process then diverges or converges outside the constraints), correct the starting point and change the penalty function so that the iteration converges and leads to reasonable results.
2. The objective function and the assumed wing mass dependence and the penalty function can be adopted differently than in the discussed example. Appropriate changes should be made if the presented example does not meet the requirements of the designed aircraft. A detailed discussion of each case requires consultation with the lab instructor.

The report should include:

- Wing geometry data
- Mass and aerodynamic characteristics of the aircraft and the minimum velocity of the aircraft

- Description of the optimisation process
- Final results, i.e. optimised wing span and chord
- Comparison between the final results and input data (initial values taken from the Aircraft Design project)

<u>Bibliography:</u>

3. Daniel P. Raymer: Aircraft Design, A conceptual Approach, AIAA 2004
4. Garret N. Vanderplaats: Numerical Optimization Techniqes for Engineering Design, McGraw-Hill, 1984